AP Statistics Chapter 16 - 17 Review

****ANSWERS ONLY** (for explanations, please come in for tutorials)**

1. a) 0.589 b) 0.0144 c) 0.9883 d) E(X) = 2.847, SD(X) = 1.3227

2. a) 0.2404 b) 0.1203 c) 0.2508 d) 0.9987 e) 1.6978

3. a) check your notes! b) E(X) = 88.35, SD(X) = 6.0259 c) 0.9734

4. a) $E(X1 + X2 + ... + X80) = \$485.60$, $SD(X1 + X2 + ... + X80) = \$10.107$ b) 0.0771

5. In the table: 0.27, 0.345, 0.385

6. a) 4 inches b) heights of boys and girls must be independent!!! c) 5.4083 inchhes d) 0.1336

7. a) E(3X) = 57, SD(3X) = 27 b) E(Y1 + Y2 + Y3) = 72, SD(Y1 + Y2 + Y3) = 8.66

   c) E(X + 4Y) = 115, SD(X + 4Y) = 21.9317

8. ???

*9. a) 839.6 grams b) 7.91 grams

*10. a) Approximately normal, mean = 30, standard deviation = 22.361 b) 0.9101 c) 82.1 minutes

**Full solutions for #9 and #10 start on the next page...*

9. Each full carton of Grade A eggs consists of 1 randomly selected empty cardboard container and 12 randomly selected eggs.

The weights of the empty cardboard containers, C, have a mean of 20 grams and a standard deviation of 1.7 grams.

The weights of the individual Grade A eggs, E, have a mean of 68.3 grams and a standard deviation of 2.23 grams.

It is reasonable to assume independence between the weights of the empty cardboard containers and the weights of the eggs. It is also reasonable to assume independence among the weights of the 12 eggs that are randomly selected for a full carton.

Let the random variable X represent the weight (in grams) of a full carton of Grade A eggs (empty cardboard container **plus** 12 randomly selected eggs).

a) What is the mean of X?

$$E(X) = E(\text{carton}) + E(\text{egg}\#1) + E(\text{egg}\#2) + \dots + E(\text{egg}\#12)$$
$$= E(C) + 12\,E(E)$$
$$= 20 + 12(68.3)$$
$$= \boxed{839.6 \text{ grams}}$$

b) What is the standard deviation of X?

You must add variances:

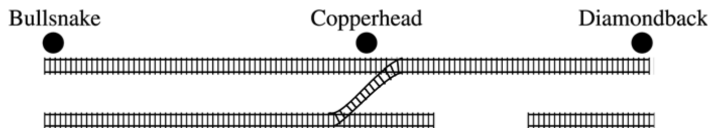
$$Var(X) = Var(C) + Var(E_1) + Var(E_2) + \dots + Var(E_{12})$$
$$= Var(C) + 12\,Var(E)$$
$$= 1.7^2 + 12\cdot(2.23^2)$$

☆ Remember: $Var = SD^2$

$$Var(X) = 62.5648 \text{ grams}$$

$$SD(X) = \sqrt{62.5648} \approx \boxed{7.91 \text{ grams}}$$

10. Flooding has washed out one of the tracks of the Snake Gulch Railroad. The railroad has two parallel tracks from Bullsnake to Copperhead, but only one usable track from Copperhead to Diamondback, as shown in the figure below. Having only one usable track disrupts the usual schedule. Until it is repaired, the washed-out track will remain unusable. If the train leaving Bullsnake arrives at Copperhead first, it has to wait until the train leaving Diamondback arrives at Copperhead.



Every day at noon a train leaves Bullsnake heading for Diamondback and another leaves Diamondback heading for Bullsnake.

Assume that the length of time, X, it takes the train leaving Bullsnake to get to Copperhead is approximately normally distributed with a mean of 170 minutes and a standard deviation of 20 minutes.

Assume that the length of time, Y, it takes the train leaving Diamondback to get to Copperhead is approximately normally distributed with a mean of 200 minutes and a standard deviation of 10 minutes.

These two travel times are independent.

a) What is the distribution of $Y - X$?
Hint: this means you must describe shape, center (mean), and spread (standard deviation) of "Y – X"

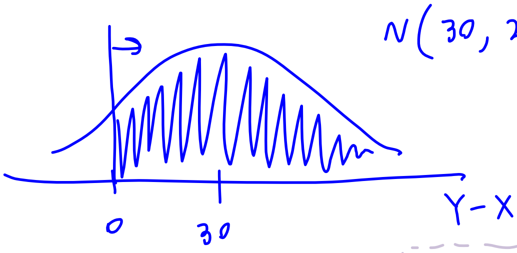
The distribution of $Y-X$ is approximately normally distributed, with...

mean: $E(Y-X) = E(Y) - E(X)$
$= 200 - 170$
$\mu_{Y-X} = 30$

St. dev: $Var(Y-X) = var(Y) + Var(X)$
$= 10^2 + 20^2$
$= 500$
$SD(Y-X) = \sqrt{500} \approx 22.361$

b) Over the long run, what proportion of the days will the train from Bullsnake have to wait at Copperhead for the train from Diamondback to arrive?
Hint: this means that Y > X... or Y – X > 0. Use the normal model and the mean & standard deviation from (a) to calculate this.

$N(30, 22.361)$



$z = \frac{obs - exp}{st.dev}$

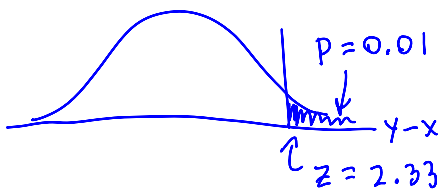
Technically: $z = \frac{(Y-X) - \mu_{Y-X}}{\sigma_{Y-X}}$

$z = \frac{0 - 30}{22.361} = -1.34$ $\leadsto$ $P(z > -1.34) = \boxed{0.9101}$ (via calculator)

THIS PROBLEM CONTINUES ON THE FOLLOWING PAGE →

c) How long should the Snake Gulch Railroad delay the departure of the train from Bullsnake so that the probability that it has to wait is only 0.01 ?

"D" = delay.



$E(Y) = 200$

$E(\text{new } X) = 170 + D$

$\Rightarrow E(Y - \text{new } X) = 200 - (170 + D)$

$= 200 - 170 - D$

$= (30 - D)$

$$z = \frac{\text{obs} - \text{mean}}{\text{st. dev}}$$

$$2.33 = \frac{0 - (30 - D)}{22.361}$$

The standard deviation of "Y – X" is still 22.361 because adding / subtracting a constant ("D") does not affect the standard deviation.

$$2.33 = \frac{D - 30}{22.361}$$

$$52.1 = D - 30$$

$$D = \boxed{82.1 \text{ minutes}}$$