Sampling Distributions

part ii: means

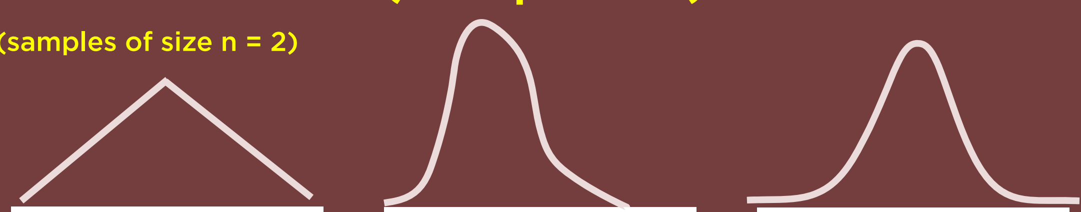
AP Statistics

Chapter 18

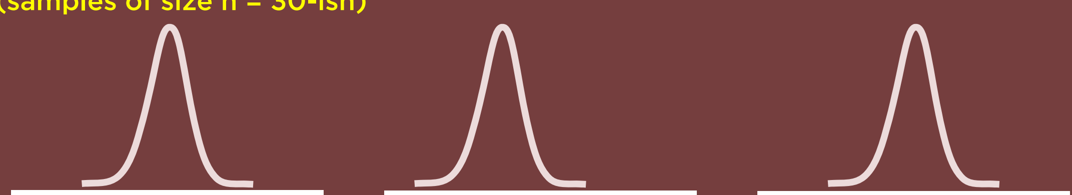
POPULATION DISTRIBUTION LOOKS LIKE:

IF WE TAKE MAAAAANY SAMPLES OF THIS SIZE,
THE SAMPLING MODEL (of sample means) WILL LOOK LIKE:

(samples of size n = 2)



(samples of size n = 30-ish)



CENTRAL LIMIT THEOREM (CLT)

When the sample size (n) is LARGE ENOUGH*, the distribution of sample means ($\bar{x}$) is **ROUGHLY NORMAL**, even when the population distribution is NOT.

*"Large enough" means "n" should be at least about 25 or 30… give or take depending on the type of distribution…

SAMPLING DISTRIBUTION FOR **MEANS**:

These are found on the formula chart!

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Again… we'll have conditions to check…

Conditions (sampling distributions with means)

1. **Randomization:**
   Random or representative sample

2. **10% condition:**
   Population should be at least 10x the sample size

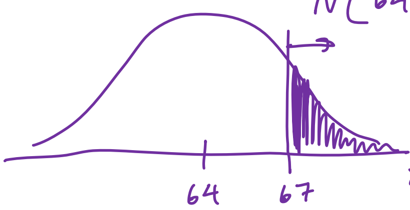
3. **Nearly Normal Condition:**
   Two ways to show this:
   1. Population is roughly normally distributed
   2. Sample size > 30 (by the Central Limit Theorem)

AP Statistics Name: NOTES Per: ______
Chapter 18 (Sampling Distributions with Means)

The Tall Female Problem

Adult female heights in the United States are roughly normally distributed with a mean height of about 64 inches, and a standard deviation of about 3 inches.

a) What is the probability that ONE randomly selected female is taller than 67 inches? (we learned these 1st semester)



x = female heights (inches)

$z = \frac{x - \mu}{\sigma}$

$= \frac{67 - 64}{3}$

$z = 1.0$

$P(x > 67)$

$= P(z > 1.0)$

$= 1 - 0.8413 = \boxed{0.1587}$

b) If we take MAAAAAANY random samples of 10 females, describe the distribution of **sample means** for these heights. → (CUSS & BS...)

The distribution of sample mean heights will be...

SHAPE: Approximately normal...

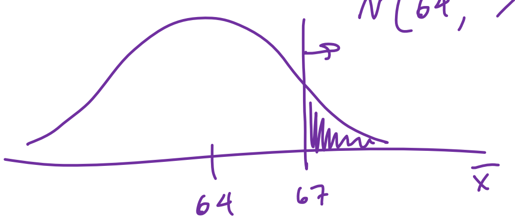
CENTER: $\mu_{\bar{x}} = \mu = 64$ inches

SPREAD: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{10}} = 0.9487$ inches

c) If we take a random sample of 10 females, what is the probability that their **mean height** is greater than 67 inches?

Since we have a random sample of females, and since female heights are roughly normally distributed, we may use the Normal model.

$N\left(64, \frac{3}{\sqrt{10}}\right)$



$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{67 - 64}{3/\sqrt{10}} = 3.16$

$P(\bar{x} > 67)$

$= P(z > 3.16)$

$= 1 - 0.9992$

$= \boxed{0.0008}$

The rabbit problem

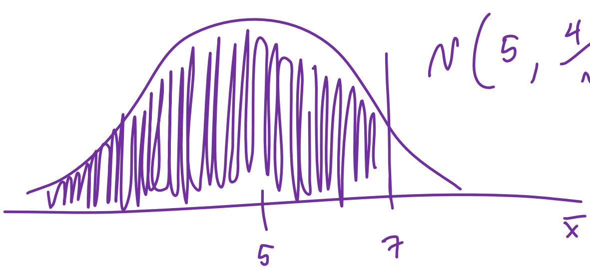
A particular breed of rabbits has a mean weight of 5 pounds, with a standard deviation of 4 pounds. However, the distribution of weights for these rabbits is skewed to the right.

a) If we wish to find the probability that ONE randomly selected rabbit weighs less than 7 pounds, can we calculate this probability using the normal model?

NO. The population of weights is non-normal, so the normal model may not be used here.

b) If we wish to find the probability that a random sample of 35 of these rabbits have a mean weight of less than 7 pounds, can we calculate this probability using the normal model? If so, calculate this probability.

Since we have a sample size of 35 (which is > 30), we may use the normal model.



$N\left(5, \frac{4}{\sqrt{35}}\right)$

$P(\bar{x} < 7)$

$= P(z < 2.96)$

$= \boxed{0.9985}$

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \frac{7-5}{4/\sqrt{35}} = 2.958$

c) If we take **ONE SAMPLE** of 80 of these rabbits, what would be the shape of the distribution of weights of the sample?

Since the weights are skewed to the right, the distribution for ONE sample will likely be skewed to the right.

d) Describe the sampling distribution of the sample mean rabbit weights for random samples of 5 rabbits.

Since the sample size is small (< 30ish), the shape will still be SKEWED to the right.

Center: $\mu_{\bar{x}} = 5$ pounds,

Spread: $\sigma_{\bar{x}} = \frac{4}{\sqrt{5}} = 1.789$ pounds

e) Describe the sampling distribution of the sample mean rabbit weights for random samples of 80 rabbits.

Since the sample size > 30, the shape will be approximately normal.

center: $\mu_{\bar{x}} = 5$ pounds

$\sigma_{\bar{x}} = \frac{4}{\sqrt{80}} = 0.447$ pounds