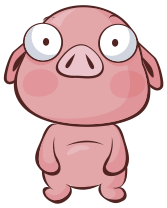
AP Statistics Name: _____ Per: _____

Obtaining P-values from Simulations

Introduction: So far in this class, we have performed hypothesis tests by using the Normal model to obtain a p-value (we then compare the p-value to α, and then come to a conclusion on whether to reject/fail to reject Ho, etc). However **SOMETIMES WE ARE UNABLE TO USE THE NORMAL MODEL**, and must obtain a p-value through alternative methods. This will be your first practice problem with such a scenario.

"Pig dice" are small plastic pig-like figures that can be rolled just like dice! If a pig is rolled and lands on its back (with all four legs in the air), this position is referred to as "naptime". Through a large number of trials, it is assumed that the true probability of rolling "naptime" is about 20%. Since these "pigs" are made of plastic and have no memory, each roll of a "pig" is independent of the next.

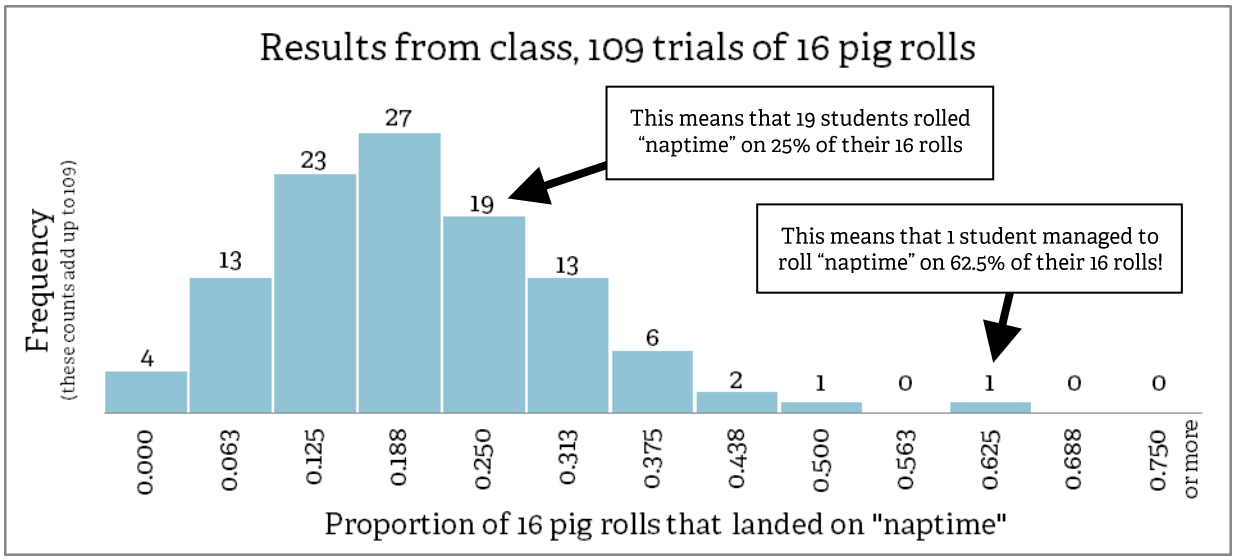


At the Podunk High School homecoming carnaval, Mu Alpha Theta sets up a game called "Lucky Naptime". In this game, the contestant rolls a pig die exactly 16 times. If at least 40% of the 16 rolls land on "naptime", then the player wins a giant stuffed animal!

During the first hour of the carnaval, 18 different contestants come to play the game (each rolling a pig die 16 times), and **NOT A SINGLE ONE OF THEM** wins a giant stuffed animal (meaning that **NONE** of them managed to roll "naptime" on at least 40% of the 16 rolls).

Suspiciously, one student then shows up with their own "special" pig die, and asks if they can use it to play the game. This student plays the game twice with their "special pig die" and somehow manages to win a giant stuffed animal BOTH GAMES. However, one of the teachers supervising this game at the carnaval – Mr. Youn – happens to be the AP Statistics teacher, and immediately confiscates the **SUSPICIOUS** pig die from the student.*

During class the next week, Mr. Youn wishes to investigate, and asks each of his 109 AP Statistics students to roll a pig die 16 times. Each of the 109 students calculates the proportion of the 16 rolls that landed "naptime", and the results of these 109 trials is shown in the histogram below:



a) How many of the 109 students managed to roll "naptime" on at least 40% (proportion of 0.400 or above) of their 16 pig rolls?

*The student was allowed to keep the two giant stuffed animals. However, since the club only had 2 stuffed animals to give away, the game had to shut down immediately afterwards. ☹

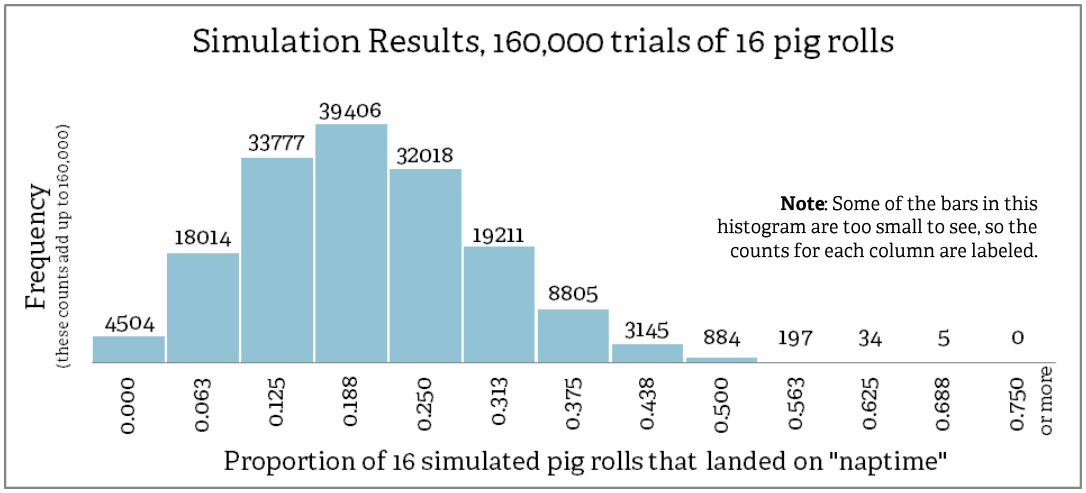
Mr. Youn wishes to perform a 1-proportion hypothesis test to see if the "suspicious" pig die that he confiscated might be **SPECIALLY WEIGHTED** to land on its back ("naptime!") **more often** than a normal pig die. Since "normal" pig dice are thought to land "naptime" on roughly 20% of all rolls, the hypotheses are:

$H_0 : p = 0.20$ $\qquad$ $H_A : p > 0.20$

where "p" represents the true proportion of rolls for this "special" pig die that land on "naptime".

Since the sample size of 16 rolls is **TOO SMALL TO USE A NORMAL MODEL APPROXIMATION**, we must use another method to estimate a p-value for this hypothesis test. One method is to use a simulation of a **LARGE NUMBER** of trials to build a probability model.

Although Mr. Youn already has data from 109 trials (via the students in his classes), he would really like to obtain data from a **MUCH LARGER** number of samples, so he uses a statistical software program on a computer to simulate 16 rolls of a pig die, and has the computer run this simulation **160,000 times**. For each of the 160,000 trials, **it was assumed that the null hypothesis was true** (in other words, that the true proportion of pig dice rolls that land "naptime" is 0.20). For each of the 160,000 trials the sample proportion of the 16 rolls that landed "naptime" was recorded. The histogram below shows the results of the 160,000 trials:



b) To obtain a sample statistic (p-hat value) for this hypothesis test, Mr. Youn rolls the "suspicious" pig die 16 times, and observes that the pig lands "naptime" on 7 of the 16 rolls, **for a sample proportion of 0.438**.

Based on this sample proportion of 0.438 **AND** based on the results of the 160,000 simulated trials shown in the graph above, **what would be the p-value for this hypothesis test**? Express your p-value as a decimal between 0 and 1, and clearly show how you calculated this value.
(Hint: Think about the definition of p-value. YOU ARE **NOT** ALLOWED TO USE THE NORMAL MODEL/Z-TABLE!!!)

c) Based on your p-value from part (b), what would you conclude about the hypothesis test for the hypotheses given at the top of this page? Use an α = 0.05 level of significance.
(reject Ho, fail to reject Ho? Is there evidence that this "special" pig die lands "naptime" MORE often than a normal pig die?)