

Scaling/Shifting with Means and Variances

remember: variance is (st. dev)²

$$E(X \pm c) = E(X) \pm c$$

$$\text{Var}(X \pm c) = \text{Var}(X)$$

$$\text{SD}(X \pm c) = \text{SD}(X)$$

$$E(aX) = a \cdot E(X)$$

$$\text{Var}(aX) = a^2 \cdot \text{Var}(X)$$

$$\text{SD}(aX) = a \cdot \text{SD}(X)$$

For any two random variables, "X" and "Y":

$$E(X \pm Y) = E(X) \pm E(Y)$$

If "X" and "Y" are independent:

ALWAYS a plus!

$$\text{Var}(X \pm Y) = \text{Var}(X) + \text{Var}(Y)$$

$$\text{SD}(X \pm Y) = \sqrt{\text{SD}(X)^2 + \text{SD}(Y)^2}$$

"X" and "Y" MUST be independent!!!

If they're NOT, then we cannot determine the variance (or standard deviation) of the combined random variable.

1. X and Y are two independent random variables with the following attributes:

$$E(X) = 11$$

$$\text{SD}(X) = 9$$

$$E(Y) = 24$$

$$\text{SD}(Y) = 5$$

Find the mean and standard deviation of each of these random variables:

a) $3X$ $E(3X) = 3(11) = \boxed{33}$
 $\text{SD}(3X) = 3 \cdot 9 = \boxed{27}$

e) $X_1 + X_2 + X_3$ (not the same as "3X"!!!)
 $E(X_1 + X_2 + X_3) = 11 + 11 + 11 = \boxed{33}$
 $\text{SD}(X_1 + X_2 + X_3) = \sqrt{9^2 + 9^2 + 9^2}$

b) $Y - 15$ $E(Y - 15) = 24 - 15 = \boxed{9}$
 $\text{SD}(Y - 15) = \boxed{5}$ ** shift does NOT affect SD!*

$$= \sqrt{3} \times 9 = \boxed{15.58}$$

c) $X + Y$ $E(X + Y) = 11 + 24 = \boxed{35}$
 $\text{SD}(X + Y) = \sqrt{9^2 + 5^2} = \boxed{10.296}$

f) $5X - 3Y$
 $E(5X - 3Y) = 5(11) - 3(24) = \boxed{-17}$

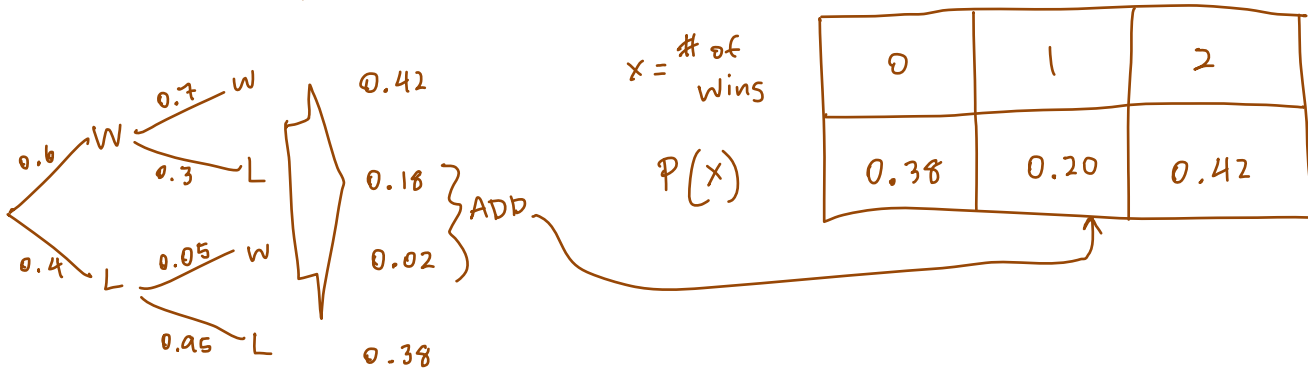
d) $X - Y$ $E(X - Y) = 11 - 24 = \boxed{-13}$
 $\text{SD}(X - Y) = \sqrt{9^2 + 5^2} = \boxed{10.296}$

$$\text{SD}(5X - 3Y) = \sqrt{(5 \times 9)^2 + (3 \times 5)^2} = \boxed{47.43}$$

ALWAYS ADD VARIANCE!

2. **The Podunk Polar Bears** (a football team) have two games left in their season (so far they are winless). Experts estimate that the team has a 60% probability of winning the first game. If they win the first game, they have a 70% chance of winning the 2nd game. Otherwise, they only have a 5% chance of winning the second game.

Construct a probability model for the number of games that the Polar Bears will win. *make a table*



The Die (Singular) Game Problem

3. You roll a die. If it comes up a 6, you win \$100. If not, you get to roll again, and if you get a 6 the second time, you win \$50. If not, you lose ☹️. Create a probability model for the amount you win at this game, and find the expected amount you'll win.

x = \$ won

100	50	0
$\frac{1}{6}$	$\frac{5}{36}$	$\frac{25}{36}$

P(x)

↑
roll a "6"

$\frac{5}{6} \cdot \frac{1}{6}$

$\frac{5}{6} \cdot \frac{5}{6}$



(if we round)

$$E(x) = \$24 \quad SD(x) = \$38$$

Does " $X_1 + X_2$ " = " $2X$ "? (continuing the Die Game Problem...)

Find the mean and standard deviation of the amount of money won if...

- a) we **double** the dollar amounts (and play the game once)

$$E(2x) = 2(24) = \boxed{\$48}$$

$$SD(2x) = 2(38) = \boxed{\$76}$$

Not the same!

- b) we **play the game twice** (without doubling the \$ amounts)

$$E(x_1 + x_2) = 24 + 24 = \boxed{\$48}$$

$$SD(x_1 + x_2) = \sqrt{38^2 + 38^2} = \boxed{\$54}$$