Hypothesis Tests *with proportions*

AP Statistics

Chapter 20

1a. (the mutating disease problem)

It is believed that the vaccine for a horrible disease has been effective on 62% of its recipients. Medical research teams have been tracking this HORRIBLE disease and are looking for evidence to see if perhaps the vaccine is now less effective than it used to be (maybe the disease has evolved?). The vaccine is administered to a random sample of 450 adults, and after following these subjects for a period of 3 months, was found to be effective on 259 of them.

a) Does this provide evidence that the vaccine has become less effective?

Hypothesis Tests

*here's our **burning** question for the day:*

Could this observed "change" be the result of chance variation?

or is the change so DRASTIC (unlikely) that it is considered "statistically significant"?

Steps:

1) Identify your parameter of interest
   (p = ___ or μ = ___)
2) Conditions to meet assumptions
3) Hypothesis statements
4) Calculations
5) Conclusion, in context

CONDITIONS:

- Random sample?
- 10% rule? (only check when sampling **WITHOUT** replacement)
- Success/failure:
  $np \geq 10$, $nq \geq 10$

If the conditions are satisfied, we may use the Normal model to conduct a

1-PROPORTION Z-TEST

HOW TO WRITE HYPOTHESES

- **Null hypothesis** – a statement of "no effect" or "no difference" or "no change"

$$H_0: \; p = p_o$$

- **Alternative hypothesis** – what we **suspect is true** or what we are **trying to show**

$$H_A: \; p > p_o$$

$$p < p_o$$

$$p \neq p_o$$

LEVEL OF SIGNIFICANCE (ALPHA)

α

- Can be any value *(but 0.05 is most common)*
- Usual values: 0.10, 0.05, 0.01
- For tests that requires a higher level of evidence, we use a **LOWER** alpha value.

Is our result **statistically significant**?

- If p-value $\leq \alpha$,

  "**reject**" the H_o.

- If p-value $> \alpha$,

  "**fail to reject**" the H_o.

α (level of significance)
Can be any value, but 0.05 is the most common.

NEVER "ACCEPT" THE H_o.

Never "**accept**" the H_o!

Never "**accept**" the H_o!

Never "**accept**" the null hypothesis!

Writing your conclusion

(ALWAYS do this!)

"Since the *p*-value **<** (**>**) α,
we **reject** (**fail to reject**) the **H_0**.

We **have** (**lack**) sufficient evidence to suggest that **[H_a in context]**"

Be sure to write H_a in context (words)!

Interpreting your p-value

(only do this when asked)

If Ho is true, this is the probability that a statistic **at least** as extreme as the observed value would occur.

(putting this in context is tricky!)

AP Statistics – Hypothesis Testing (1-proportion z-tests)

1. It is believed that the vaccine for a horrible disease has been effective on 62% of its recipients. Medical research teams have been tracking this HORRIBLE disease and are looking for evidence to see if perhaps the vaccine is now less effective than it used to be (maybe the disease has evolved?). The vaccine is administered to a random sample of 450 adults, and after following these subjects for a period of 3 months, was found to be effective on 259 of them.

   a) Does this provide evidence that the vaccine has become less effective?

   ↖ hypothesis test! ↖ direction for H_A

1-proportion z-test

p = true proportion of all adults on whom the vaccine is effective

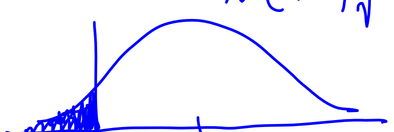
$H_0: p = 0.62$

$H_A: p < 0.62$

$$z = \frac{\text{statistic} - \text{parameter}}{\text{SD of stat}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

(obs. = statistic; mean = parameter, $\mu_{\hat{p}}$; SD = $\sigma_{\hat{p}}$)

$$N\left(0.62, \sqrt{\frac{.62(1-.62)}{450}}\right)$$



$$= \frac{\frac{259}{450} - 0.62}{\sqrt{\frac{0.62(1-0.62)}{450}}}$$

$z = -1.9424...$

p-value = 0.0260

Using $\alpha = 0.05$

Conditions for inference:

- We have a random sample of 450 adults, which is surely less than 10% of all adults.
- Normality:
  $np_0 = 450(0.62) = 279 \geq 10$ ✓
  $nq_0 = 450(1-0.62) = 171 \geq 10$ ✓

☆ Since our p-value < α, we REJECT H_0.

We have evidence that the vaccine has become less effective.

   b) Carefully interpret the meaning of the p-value that was calculated in part (a).

p = 0.026

If the vaccine is truly 62% effective, then the probability of it being effective on fewer than 259 of 450 patients due to random sampling variation (chance) is about 0.0260.

2. According to the Association of American Medical Colleges, only 46% of medical school applicants were admitted to a medical school in the fall of 2006. Two years later, a random sample of 180 medical school applicants was taken, and 77 were admitted to medical school. $\hat{p} = \frac{77}{180} \approx 0.4278$

a) Does this data provide evidence of a change in the acceptance rate for medical schools?

(evidence ← hypothesis test!; change ← no specific direction... so 2-sided H_A.)

1-proportion z-test

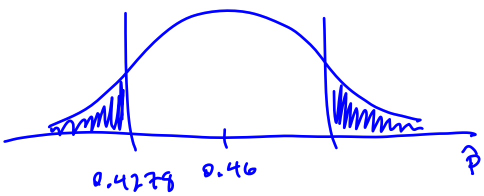
p = true proportion of medical school applicants who are admitted.

$H_0: p = 0.46$

$H_A: p \neq 0.46$

Conditions: we have a random sample of medical school applicants, and 180 is surely < 10% of all applicants.

- Normality:
$np_0 = 180(0.46) = 82.8$
$nq_0 = 180(0.54) = 97.2$ > 10 ✓



$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{\frac{77}{180} - 0.46}{\sqrt{\frac{0.46(1-0.46)}{180}}}$$

$z = -0.8674$

p-value $= 0.3857$

(make sure you double the tail probability!)

☆ Using $\alpha = 0.05$:

Since $p > \alpha$, we fail to reject H_0.

We lack sufficient evidence of a change in the proportion of students that are admitted to medical school.

b) Interpret the meaning of your p-value from part (a) in context.

If the true medical school acceptance rate is 46%, then the probability of observing a sample acceptance rate at least as extreme as the one in this study $\left(\frac{77}{180}\right)$ as a result of chance variation is about 39%. (YES, THESE ARE PAINFUL)