

## AP Statistics

### P-values and Power Practice

- Attitudes** The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures students' attitudes toward school and study habits. Scores range from 0 to 200. The mean score for U.S. college students is about 115. A teacher suspects that older students have better attitudes towards school. She gives the SSHA to an SRS of 45 of the more than 1000 students at her college who are at least 30 years of age. The sample mean SSHA score was 125.7 and the sample standard deviation was 29.8. A significance test yields a P-value of 0.0101. Interpret the P-value in context.
- Anemia** Hemoglobin is a protein in red blood cells that carries oxygen from the lungs to body tissues. People with less than 12 grams of hemoglobin per deciliter of blood (g/dl) are anemic. A public health official in Jordan suspects that Jordanian children are at risk of anemia. He measures a random sample of 50 children. Their sample mean hemoglobin level was 11.3 g/dl and the sample standard deviation was 1.6 g/dl. A significance test yields a P-value of 0.0016. Interpret this P-value in context.
- Error probabilities** You read that a statistical test at significance level  $\alpha = 0.05$  has power 0.78. What are the probabilities of Type I and Type II errors for this test?
- Error probabilities** You read that a statistical test at the  $\alpha = 0.01$  level has probability 0.14 of making a Type II error when a specific alternative is true. What is the power of the test against this alternative?
- Power** A drug manufacturer claims that fewer than 10% of patients who take its new drug for treating Alzheimer's disease will experience nausea. To test this claim, a significance test is carried out of

$$H_0: p = 0.10$$

$$H_A: p < 0.10$$

You learn that the power of this test at the 5% significance level against the alternative  $p = 0.08$  is 0.64.

- Explain in simple language what "power = 0.64" means in this setting.
  - You could get higher power against the same alternative with the same  $\alpha$  by increasing the number of measurements you make (a.k.a., increasing the sample size). Explain how this would increase the power of the test.
  - If you decide to use  $\alpha = 0.01$  in place of  $\alpha = 0.05$ , with no other changes in the test, will the power increase or decrease? Justify your answer.
- Luke's Lab Partner(s)** For his statistics project, Luke is performing a hypothesis test to compare the proportion of people in Austin that have tattoos to the proportion in the rest of the United States (the true proportion of U.S. adults with a tattoo is believed to be 21%, according to a recent Harris Poll). Based on a survey that Luke conducts in downtown Austin, he calculates a z-statistic of +1.40.
    - Since Luke believes Austin to be "weird" and thinks that the proportion of Austin adults with a tattoo will be greater than that for the rest of the country, his alternative hypothesis is " $H_a: p > 0.21$ ". Based on the z-score that he calculated and his alternative hypothesis, calculate the p-value for Luke's hypothesis test.
    - Luke's lab partner, Ciara, thinks that Austin adults will have a lower proportion of tattoos than the rest of the States, so she sets her alternative hypothesis as " $H_a: p < 0.21$ ". Based on the same z-score of 1.40 (since she used the same data as Luke), calculate the p-value for Ciara's hypothesis test.
    - Marie is a friend of Luke and Ciara, and while she is not even taking Statistics, decides to join in on the fun. Marie decides to use the same data to perform a two-tailed test (so her alternative hypothesis is " $p \neq 0.21$ "). Using the same z-score as Luke and Ciara, calculate the p-value for Marie's hypothesis test.

7. **More Power** Remember that the power of a test is defined as the probability of rejecting a false null hypothesis. Suppose that the null hypothesis of a test is  $H_0: p = 0.50$ .  
(*HINT: we are MOST likely to reject a false  $H_0$  – and thus get higher power – with an actual proportion that agrees strongly with the  $H_A$* )
- i) Suppose  $H_A: p > 0.50$ , and that  $H_A$  is true. For a fixed sample size and significance level  $\alpha$ , the power of the test will be greatest if the actual proportion is which of the following?  
a) 0.40                  b) 0.48                  c) 0.52                  d) 0.60                  e) 0.64
- ii) Suppose  $H_A: p < 0.50$ , and that  $H_A$  is true. For a fixed sample size and significance level  $\alpha$ , the power of the test will be greatest if the actual proportion is which of the following?  
a) 0.40                  b) 0.48                  c) 0.52                  d) 0.60                  e) 0.64
- iii) Suppose  $H_A: p \neq 0.50$ , and that  $H_A$  is true. For a fixed sample size and significance level  $\alpha$ , the power of the test will be greatest if the actual proportion is which of the following?  
a) 0.40                  b) 0.48                  c) 0.52                  d) 0.60                  e) 0.64