AP Statistics – Inference with Two Proportions

Conditions for inference with two proportions (both intervals AND tests):

1. **INDEPENDENT RANDOM SAMPLES**
   the two groups should be randomly selected
   and
   the two groups must be independent of each other!

2. **10% CONDITION**
   Must be true for BOTH groups!

3. **SUCCESS/FAILURE CONDITION**
   $n_1\hat{p}_1 \geq 10$ $\quad n_2\hat{p}_2 \geq 10$
   $n_1\hat{q}_1 \geq 10$ $\quad n_2\hat{q}_2 \geq 10$

HOWEVER
If we have a randomized experiment: Check for random assignment of subjects to treatments INSTEAD of conditions 1 and 2.

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1. **THE RED PAINT PROBLEM** Independent random samples of drivers of 80 cars that are painted red and 267 non-red cars in a certain large city are surveyed to see how many of them have received speeding citations in the past 90 days. Of the 80 red cars, 19 have received tickets, and of the 267 non-red cars, 49 have received tickets. A certain support group claims that drivers of red cars are being unfairly targeted. Do they have evidence to support their claims?

Construct a 95% confidence interval for the difference in proportions of the two groups of cars that receive speeding citations.

p_1 = true proportion of RED cars that receive tickets
p_2 = true proportion of NON-RED cars that receive tickets

Conditions:
- We are given that we have independent and random samples of red and non-red cars
- In a large city, it is reasonable that there are at least 800 red cars and 2670 non-red cars
- Success/failure (Normality):
  Red: 19 got tickets, 61 did not.
  Non-red: 49 improved, 218 did not.
  These are all ≥ 10

We may use the normal model to construct a...

2-proportion z-interval

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$\left(\frac{19}{80} - \frac{49}{267}\right) \pm 1.96 \sqrt{\frac{\frac{19}{80}\left(1-\frac{19}{80}\right)}{80} + \frac{\frac{49}{267}\left(1-\frac{49}{267}\right)}{267}} \Rightarrow \text{serious calculator crunching} \Rightarrow (-0.0502, +0.1582)$$

We are 95% confident that the true DIFFERENCE in proportions of cars (red – nonred) that receive speeding tickets is between -0.0502 and 0.1582

2. **GASTRIC FREEZING** was once a recommended treatment for ulcers in the upper intestine. This process involved putting the patient under anesthesia, putting a balloon down into their stomach, and filling the balloon with FREEZING COOLANT. For years this treatment was considered effective against stomach ulcers, until a health insurance company decided to conduct a clinical trial.

For this clinical trial, 160 patients with ulcers were randomly divided into two groups by flipping a fair coin for each patient. 28 of the 82 patients who received the freezing treatment showed improvement, while 30 of the 78 patients that received a placebo (?!?) improved.

a) Does this study provide evidence that gastric freezing is more effective in treating ulcers than a placebo?

p_1 = true proportion of ulcer patients that improve with freezing treatment

p_2 = true proportion of ulcer patients that improve with a placebo

Conditions:

- The patients were randomly assigned to one of the two treatment groups by coin flip (so we have independent groups).
  DO NOT CHECK FOR RANDOM SAMPLES OR 10% IN AN EXPERIMENT!!!
- Success/failure (Normality):
  Freezing: 28 patients improved, 54 did not.
  Placebo: 30 improved, 48 did not.
  These are all ≥ 10

Hypotheses:

$H_0: p_1 = p_2$ OR $p_1 - p_2 = 0$

$H_A: p_1 > p_2$ OR $p_1 - p_2 > 0$

Pool the data!!!

$$\hat{p}_c = \frac{28+30}{82+78} = \frac{58}{160}$$

"C" for combined or "pooled"

We may use the normal model to conduct a...

2-proportion z-test

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\frac{\hat{p}_c\hat{q}_c}{n_1} + \frac{\hat{p}_c\hat{q}_c}{n_2}}} = \frac{\left(\frac{28}{82} - \frac{30}{78}\right) - 0}{\sqrt{\frac{\frac{58}{160}\left(1-\frac{58}{160}\right)}{82} + \frac{\frac{58}{160}\left(1-\frac{58}{160}\right)}{78}}}$$

$z = -0.5676$ p-value $= 0.7148$ (We'll use $\alpha = 0.05$)

Since our p-value is > alpha, we FAIL to reject Ho.
We LACK sufficient evidence that gastric freezing is more effective in treating stomach ulcers than a placebo.

b) Carefully interpret the meaning of the p-value that you calculated in part (a).

If there is TRULY no difference in the effectiveness of the two treatments, then the probability of observing a difference at least as large as the one in this study is about 71.48%.