

**Part I – Free Response**

1. A production factory in Singapore produces thousands of widgets each day, which have a mean weight of 100 ounces and a standard deviation of 15 ounces. The distribution of weights of these widgets is strongly skewed to the lower weights. A quality control engineer selects a random sample of widgets and measures their weights.
  - a) If quality control were to select a large number of samples (read: maaaaaaaaaaaaany samples) of size 12, describe the sampling distribution of sample mean weights for random samples of 12 widgets.  
*(hint: you need to describe the the mean, standard deviation, and likely shape)*
  - b) If a random sample of 80 widgets is selected, describe the likely shape of the distribution of weights for that one sample.
  - c) A quality control engineer will select a random sample of 80 widgets to determine if the manufacturing process is within specifications. Describe the distribution of sample mean weights for random samples of 80 widgets. *(hint: you need to describe the the mean, standard deviation, and likely shape)*
  - d) If a random sample of 80 widgets has a mean weight of more than 105 ounces, then the production process will be halted and recalibrated. Based on the mean and standard deviation that were calculated in part (c), what is the probability that this will occur?
  
2. Big Town Fisheries recently stocked a new lake in a city park with 2,000 fish of various sizes. The distribution of the lengths of these fish is approximately normal.

Big Town Fisheries claims that the mean length of the fish is 8 inches. If the claim is true, which of the following would be more likely? Justify your answer. *(You may wish to use drawings/diagrams to help justify your answer)*

  - i. A random sample of 15 fish having a mean length that is greater than 10 inches
  - or
  - ii. A random sample of 50 fish having a mean length that is greater than 10 inches
  
3. Providence Memorial Hospital is conducting a blood drive because its supply of group O blood is low, and it needs donors of group O blood.
  - a) Suppose the Greater New York Blood Program wishes to estimate the true proportion of area adults that have type O blood by using a confidence interval based on the results of a survey. What is the minimum number of adults that must be surveyed in order to guarantee that their margin of error is no more than 4.5% with a confidence level of 98%?
  - b) According to data provided by the Greater New York Blood Program, forty-five percent of adults have group O blood (which we will assume to be true). For random samples of 400 donors, describe the mean and standard deviation of the sample proportion of donors that have group O blood.
  - c) It is estimated that the hospital will need at least 160 units of group O blood from the 400 donors. Assuming that 45% of all adults have group O blood, use the mean and standard deviation calculated in part (b) to find the probability that at least 160 of the 400 donors will have group O blood. Be sure to verify any conditions that are necessary for your analysis.

4. A local news organization wishes to estimate the proportion of all Austin households that have experienced some sort of crime by constructing a 95% confidence interval based on survey data.
- In the context of this situation, interpret the 95% confidence level. (*interpret the LEVEL!*)
  - In a random sample of 300 Austin households, 68 report that they had experienced some sort of crime. Construct and interpret a 95 confidence interval to estimate the proportion of Austin households that have experienced some sort of crime.
  - The Gallup Organization conducted a national study on crime victimizations and reports that 25% of all households experience some sort of crime. Based on this interval, is there evidence to suggest that the proportion of Austin households who experienced some sort of crime is less than the national proportion?
  - Suppose that another media organization wishes to conduct a similar survey in Austin. What is the minimum number of households that they would need to survey in order to guarantee a margin of error of no more than  $\pm 2\%$  with a confidence level of 93%?

## Part II – Multiple Choice

- \_\_\_ 5. Create a sample of students by randomly selecting the first student of the first 20 students in the student directory and then selecting every 20<sup>th</sup> name after that. This process is a...
- simple random sampling
  - cluster sampling
  - stratified sampling
  - systematic sampling
  - convenience sampling
- \_\_\_ 6. A sample of size 49 is drawn from a normal population with a mean of 63 and a standard deviation of 14. What are the mean and standard deviation of the sampling model of sample means ( $\bar{X}$ )?
- $\mu = 9, \sigma = 2$
  - $\mu = 1.286, \sigma = 3.5$
  - $\mu = 63, \sigma = 2$
  - $\mu = 9, \sigma = 14$
  - $\mu = 63, \sigma = 0.286$
- \_\_\_ 7. In general, how does multiplying the sample size by 9 affect the width of a confidence interval?
- The width becomes three times as large.
  - The width of the interval becomes two times as large.
  - The width of the interval becomes half as large.
  - The width of the interval becomes one-third as large.
  - We need to know the sample size to be able to determine the effect.
- \_\_\_ 8. A researcher plans a study to examine the depth of belief in God among the adult population. He obtains a simple random sample of 100 adults as they leave church one Sunday morning. All but one of them agrees to participate in the survey. Which of the following are true statements?
- Proper use of chance as evidenced by the simple random sample makes this a well-designed survey.
  - The high response rate makes this a well-designed survey.
  - Selection bias makes this a poorly designed survey.
- I only
  - II only
  - III only
  - I and II
  - None of these is true

- \_\_\_ 9. A 95% confidence interval for *the proportion of female athletes in college programs* is constructed based on data from 30 randomly selected coeducational colleges. If the confidence interval is (0.38, 0.52), we can say
- A) 95% of the time, colleges have between 38% and 52% female athletes.
  - B) 95% of colleges have an average of 45% female athletes.
  - C) If 100 samples were taken and a 95% confidence interval was computed for each, 5 of the intervals would be between 38% and 52%.
  - D) If this procedure were repeated *many* times, 95% of the sample proportions would be between 38% and 52%.
  - E) If this procedure were repeated *many* times, 95% of the resulting confidence intervals would contain the true proportion of female athletes in coeducational colleges.
- \_\_\_ 10. The population {2, 3, 5, 7} has mean  $\mu = 4.25$  and standard deviation  $\sigma = 1.92$ . When sampling with replacement, there are 16 different possible ordered samples of size 2 that can be selected from this population. The mean of each of these 16 samples is computed. For example, 1 of the 16 samples is (2, 5), which has a mean of 3.5. The distribution of the 16 sample means has its own mean  $\mu_{\bar{x}}$  and its own standard deviation  $\sigma_{\bar{x}}$ . Which of the following statements is true?
- A)  $\mu_{\bar{x}} = 4.25$  and  $\sigma_{\bar{x}} = 1.92$
  - B)  $\mu_{\bar{x}} = 4.25$  and  $\sigma_{\bar{x}} > 1.92$
  - C)  $\mu_{\bar{x}} = 4.25$  and  $\sigma_{\bar{x}} < 1.92$
  - D)  $\mu_{\bar{x}} > 4.25$
  - E)  $\mu_{\bar{x}} < 4.25$
- \_\_\_ 11. The distribution of SAT Math scores of students taking Statistics at a very large university is *skewed right* with a mean of 625 and a standard deviation of 44.5. If random samples of 100 students are repeatedly taken, which give the sampling model, which statement best describes this sampling model of sample means?
- A) Roughly symmetric and unimodal with a mean of 625 and standard deviation of 44.5.
  - B) Roughly symmetric and unimodal with a mean of 625 and standard deviation of 4.45.
  - C) Skewed to the right with a mean of 625 and standard deviation of 44.5.
  - D) Skewed to the right with a mean of 625 and standard deviation of 4.45.
  - E) No conclusion can be drawn since the population is not normally distributed.
- \_\_\_ 12. The distribution of SAT Math scores of students taking Statistics at a very large university is *skewed right* with a mean of 625 and a standard deviation of 44.5. If you select a single random sample of 100 students and create a graph of their SAT Math scores, which of the following describes the likely shape of the distribution of 100 scores?
- A) skewed left
  - B) skewed right
  - C) roughly normal
  - D) uniform
  - E) all of the above

13. An urn contains exactly three balls numbered 1, 2, and 3, respectively. Random samples of two balls are drawn from the urn with replacement. The average,

$$\bar{X} = \frac{X_1 + X_2}{2},$$

where  $X_1$  and  $X_2$  are the numbers on the selected balls, is recorded after each drawing. Which of the following describes the sampling distribution of  $\bar{X}$ ? (*Hint: Make a plot of means for every possible combination of 2 numbers...*)

(A)

$\bar{X}$	1	1.5	2	2.5	3
Probability	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

(B)

$\bar{X}$	1	1.5	2	2.5	3
Probability	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{9}$

(C)

$\bar{X}$	1	1.5	2	2.5	3
Probability	0	0	1	0	0

(D)

$\bar{X}$	1	1.5	2	2.5	3
Probability	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

(E) It cannot be determined from the information given.

14. In the most recent reading, Gallup found 44% in favor of stricter laws. For their results, one can say with 95% confidence that the maximum margin of sampling error is  $\pm 4$  percentage points. Which of these statements regarding the Gallop poll about the laws governing the sale of firearms is/are true?
- A) In the sample of 1018 U.S. adults, somewhere between 40% and 48% of them were in favor of stricter laws governing the sale of firearms.
  - B) We are 95% confident that 44% of all U.S. adults are in favor of stricter laws governing the sale of firearms.
  - C) We are 95% confident that between 40% and 48% of all U.S. adults are in favor of stricter laws governing the sale of firearms.
  - D) We know that between 40% and 48% of all U.S. adults are in favor of stricter laws governing the sale of firearms.
  - E) 95% of all U.S. adults are in favor of stricter laws governing the sale of firearms.

*Note for #15: Technically, we have not done confidence intervals with means yet, but the same concepts apply to interpreting the meaning of the interval and/or confidence level as proportions.*

15. Monthly rent was determined for each apartment in a random sample of 100 apartments. The sample mean was \$820 and the sample standard deviation was \$25. An approximate 95 percent confidence interval for the true mean monthly rent for the population of apartments from which this sample was selected is (\$815, \$825). Which of the following statements is a correct interpretation of the 95 percent confidence level?
- A) In this population, about 95 percent of all rental prices are between \$815 and \$825.
  - B) In this sample, about 95 percent of the 100 rental prices are between \$815 and \$825.
  - C) In repeated sampling, the method produces intervals that include the population mean approximately 95 percent of the time.
  - D) In repeated sampling, the method produces intervals that include the sample mean approximately 95 percent of the time.
  - E) There is a probability of 0.95 that the true mean is between \$815 and \$825.