AP Statistics – Inference with Matched Pairs

"IS CAFFEINE DEPENDENCE REAL?" Researchers designed an experiment to study the effects of caffeine withdrawal. They recruited 11 volunteers who were diagnosed as being caffeine dependent to serve as subjects. Each subject was barred from coffee, colas, and other substances with caffeine for the duration of the experiment. During one two-day period, subjects took capsules containing their normal caffeine intake. During another two-day period, they took placebo capsules. The order in which subjects took caffeine and the placebo was randomized. At the end of each two-day period, a test for depression was given to all 11 subjects. Researchers wanted to know whether being deprived of caffeine would lead to an increase in depression.

The table below contains data on the subjects' scores on a depression test. Higher scores show more symptoms of depression.

Results of a caffeine-deprivation study

Subject	1	2	3	4	5	6	7	8	9	10	11
Caffeine	5	5	4	3	8	5	0	0	2	11	1
Placebo	16	23	5	7	14	24	6	3	15	12	0

Differences: (Caff − plac) −11 −18 −1 −4 −6 −19 −6 −3 −13 −1 +1

☆ For PAIRED data, we are only concerned with the distribution of differences.

a) Do the data from this study provide statistical evidence at the 5% level of significance that caffeine deprivation leads to an increase in depression?

(in other words, does no caffeine → higher depression?)

Paired t-test

μ_D = true mean difference in depression score, $\mu_C - \mu_P$ (caffeine, placebo)

$H_0: \mu_D = 0 \leftarrow \mu_C = \mu_P$

$H_A: \mu_D < 0 \leftarrow \mu_C < \mu_P$

Conditions:
- the order of treatments was randomly assigned to the volunteers.
- Sample data:

The graph of differences shows skewness, however normality should still be reasonable.

Sample data:

$\bar{x}_D = -7.36$

$s_D = 6.918$

$n = 11$

$df = 11 - 1 = 10$

$$t = \frac{\bar{x}_D - 0}{s_D/\sqrt{n}} = \frac{-7.36 - 0}{6.918/\sqrt{11}}$$

(from calculator:)

$t = -3.53$

$p\text{-value} = 0.0027$

$\alpha = 0.05$

☆ Since $p < \alpha$, we reject H_0. We have evidence that caffeine deprivation leads to a higher mean depression score.

b) Use a 90% confidence interval to estimate the true mean increase in depression scores that results from being deprived of caffeine.

Paired t-interval

$df = 10$

$$\bar{x}_D \pm t^* \frac{s_D}{\sqrt{n}}$$

$$-7.3636 \pm 2.160 \times \frac{6.918}{\sqrt{11}}$$

$(-11.144, -3.583)$

We are 90% confident that the true mean difference in depression score (caffeine − placebo) is between −11.144 & −3.583.